MEDT8007
Simulation methods in ultrasound imaging

Dynamic objects

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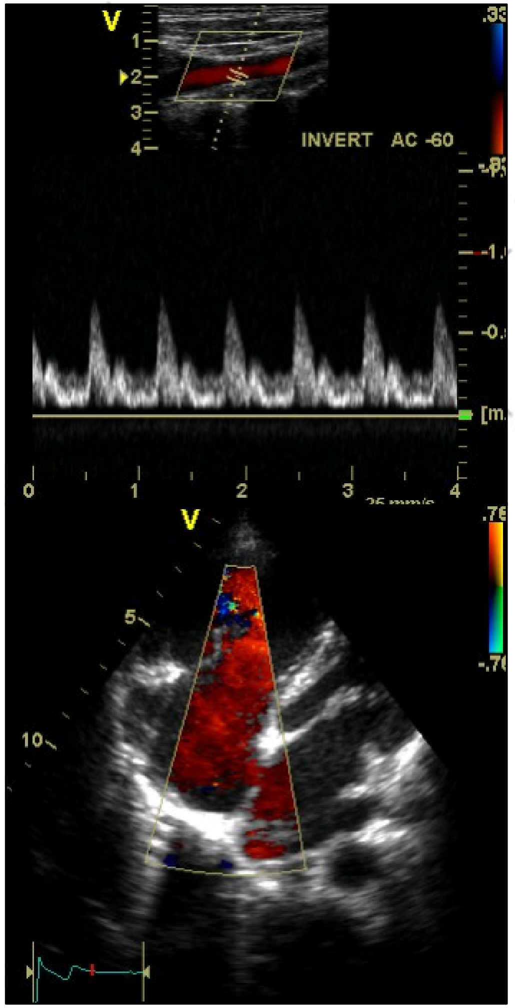
Lecture outline

- Simple simulation of Doppler signals
  - Complex Gaussian signal model
  - Exercise 1
- Simulation of blood flow in Field II
  - Requirements and pros / cons
  - Exercise 2
- Analytic blood flow models
- Computational blood flow models
- Simulation of tissue movement using FUSK

Simulation of blood flow signal

Blood flow imaging modalities

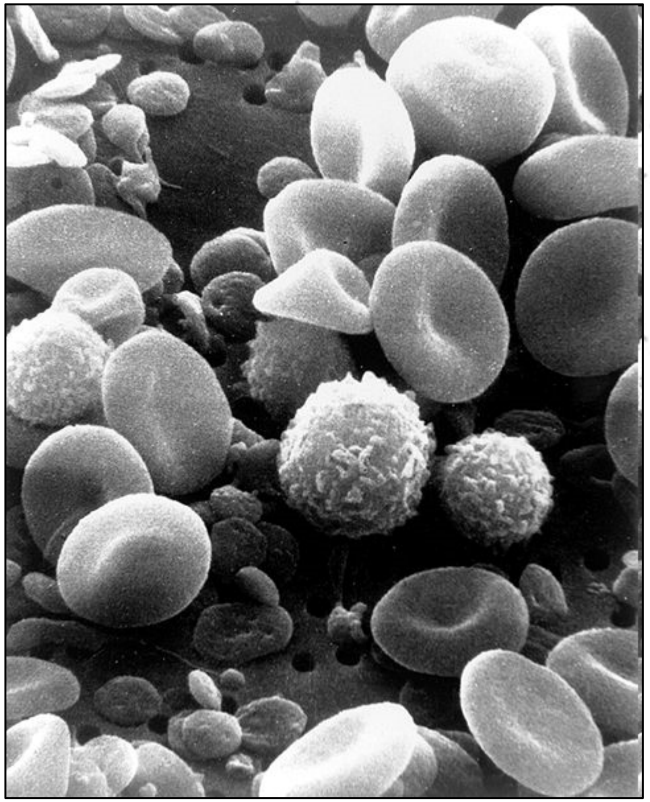
- CW- / PW-Doppler
  - Estimates the complete spectrum of velocities within a small region of interest
  - Allows for in-depth analysis of blood flow characteristics
- Color flow imaging
  - Estimates the mean velocity and direction of blood in a two- or three-dimensional region of interest
  - Allows for easy identification of abnormalities manifested in blood flow patterns
- Contrast agent imaging
  - Improved signal-to-noise ratio
  - Microcirculation imaging



Blood constituents

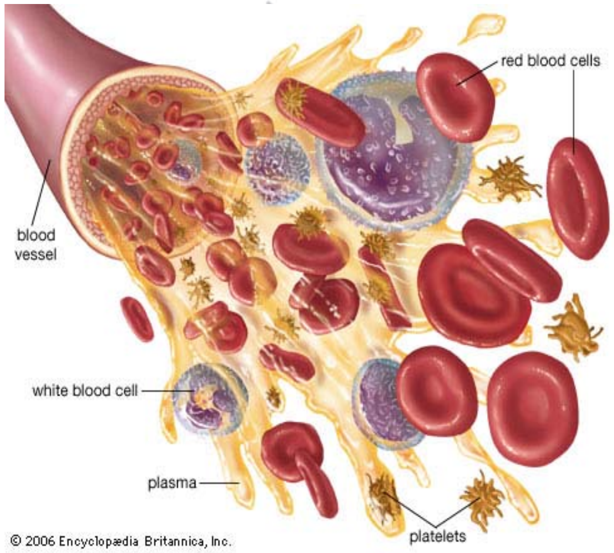
- Blood consists mainly of three constituents
  - Red blood cells (erythrocytes), ~7um diameter
  - White blood cells (leukocytes), ~14um
  - Platelets, 1/1000 red blood cell surface

- There is typically 5 litres of blood in an adult human, ~8% of body weight
  - 1 000 000 red blood cells per microliter
  - 4 - 10 000 white blood cells per microliter
  - 250 - 450 000 platelets per microliter



Scattering properties of Blood

- Scattering size is much smaller the wavelength in ultrasound
  - Typical size ~6-8 μm
- Mainly Rayleigh scattering characteristics, uniform scattering which follows a power law of f^4
  - Higher frequencies → substantially higher backscattering
- Blood scattering properties are complex and depend on *hematocrit*, turbulence, shear rates, …

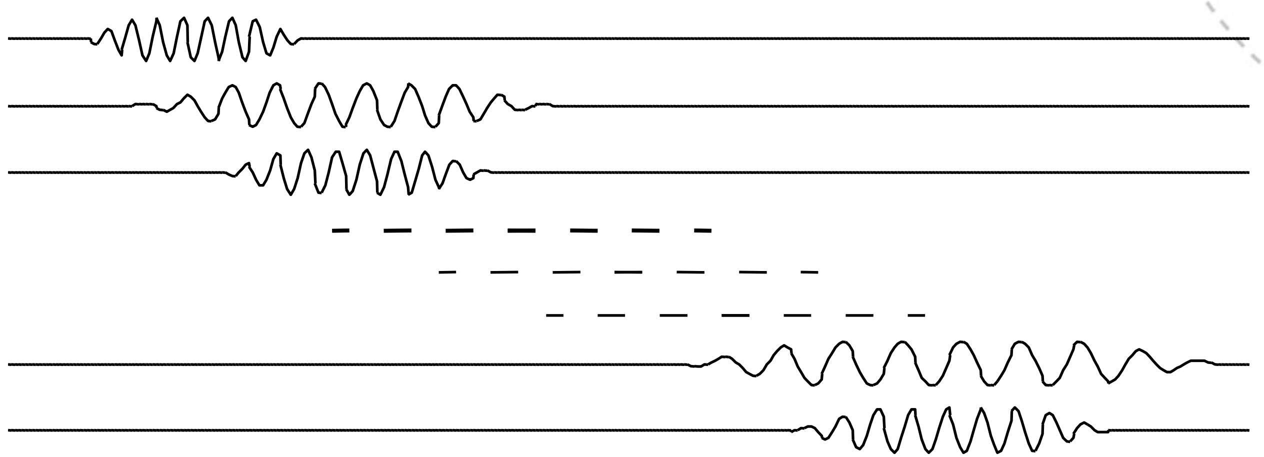


Blood scattering models

- A large collection of particle objects
  - Superposition can be applied to sum the backscattered wavelets from each red blood cell
- A random continuum
  - Scattering volumes consists of many red blood cells which has a given fluctuation in density and compressibility
- Hybrid model incorporation both the above
  - More accurate…more complex

None of the current models can predict all the scattering properties of blood, especially for low or high values of hematocrit, and in presence turbulence and shear forces

Signal from a large number of red blood cells add up to a Gaussian random process



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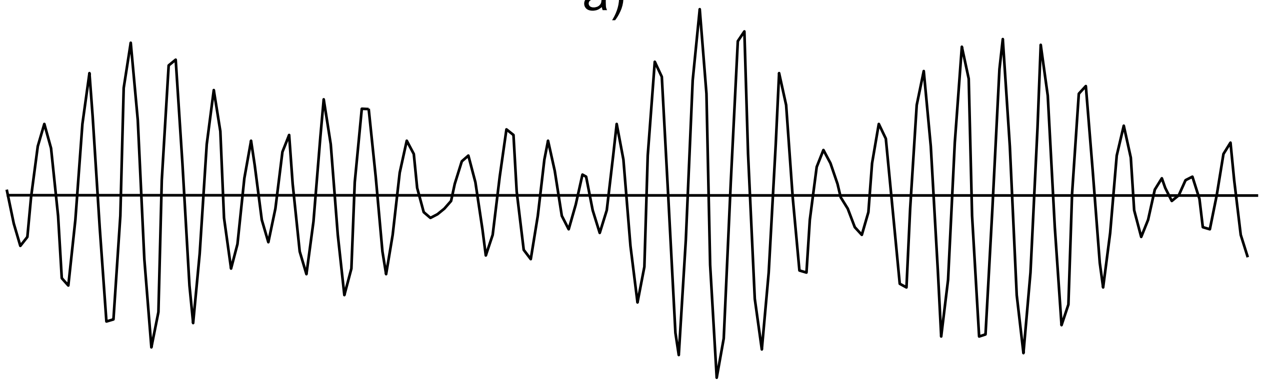


b)

Definition of Complex Gaussian process

$$p(z) = \pi^{-N} \left|\mu\right|^{-1} e^{-z^T \mu^{-1} z}$$

signal vector $z = z(1), z(2), \ldots, z(N)$

Covariance matrix $\mu = \left\langle z^{\mathrm{T}} z \right\rangle = \left\{ \left\langle z(k)^* \cdot z(n) \right\rangle \right\}_{k,n}$

$\langle - \rangle$ is expected value (ensemble average)

$$Note\ that: \left\langle z(k) \cdot z(n) \right\rangle = 0$$

Stationary complex Gaussian process

Autocorrelation function

$$R(m) \equiv \langle z(n)^* z(n+m) \rangle \quad m = 0, \pm 1, \pm 2, \ldots\ldots$$

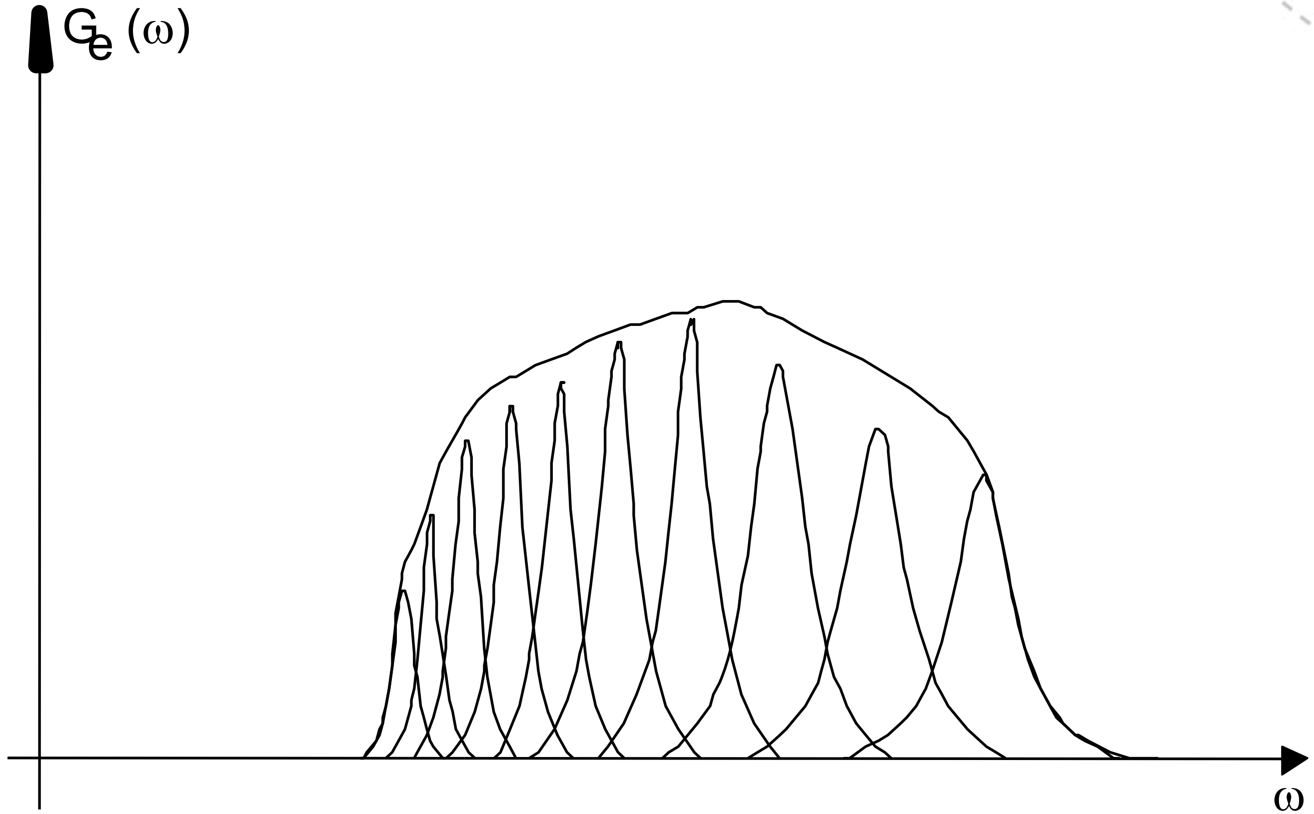
Power spectrum

$$G(\omega) \equiv \sum_m R(m) e^{-i\omega m} \quad ; \quad -\pi < \omega < \pi$$

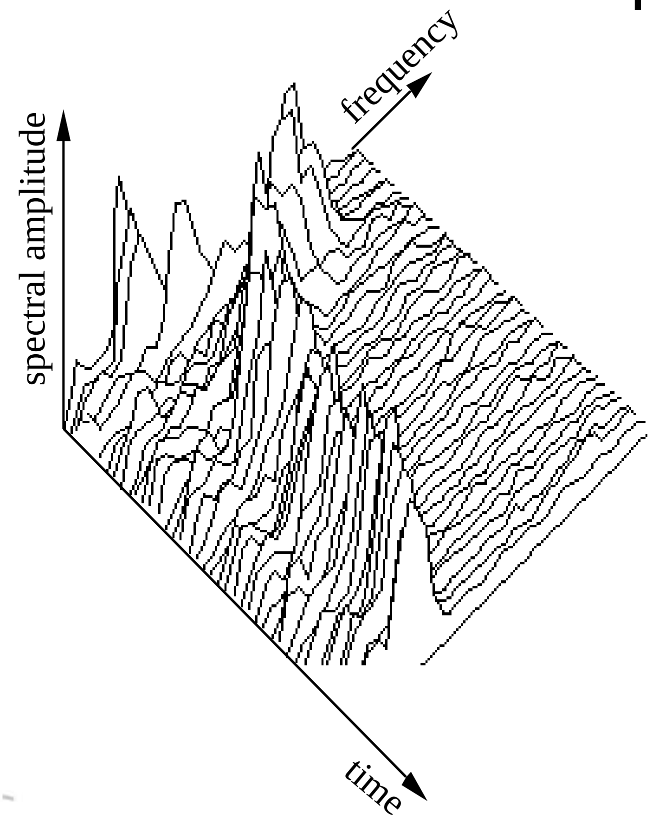
Autocorrelation function = coefficients in Fourier series of G

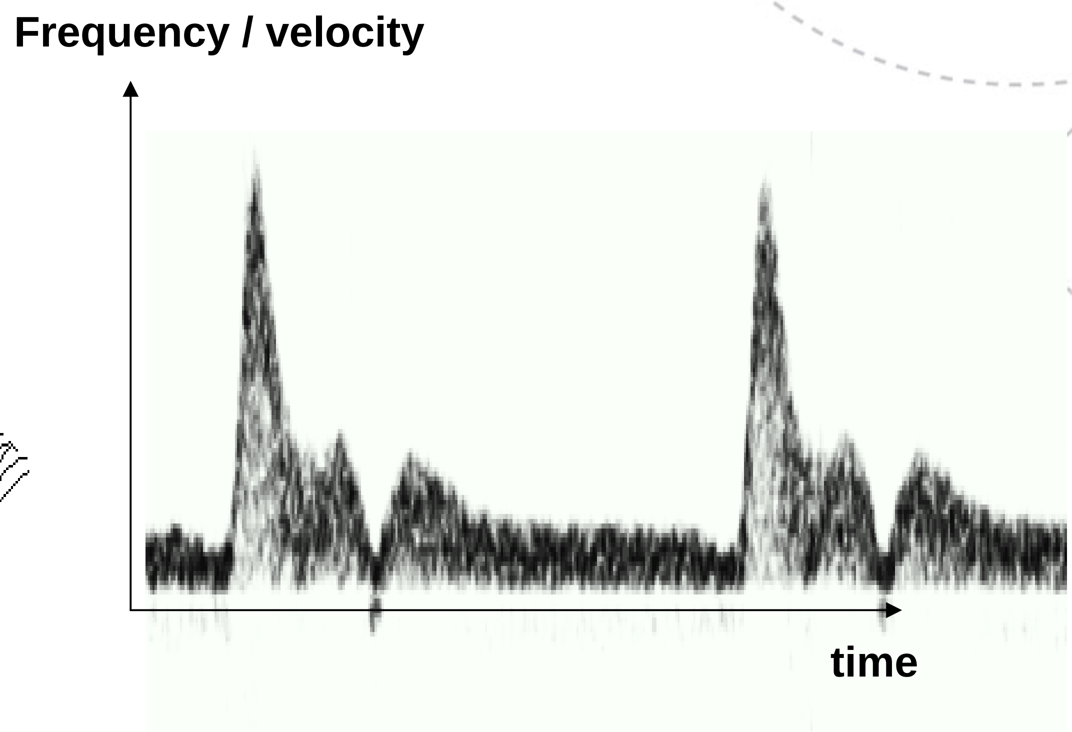
$$R(m) = \frac{1}{2\pi} \int_{-\pi}^{\pi} d\omega G(\omega) e^{i\omega m}$$

The power spectrum of the Doppler signal represents the distribution of velocities within the blood vessel



The Doppler spectrum (sonogram)





Simulation of a Gaussian stationary process

Generating realizations of a complex Gaussian process

1. Generate complex Gaussian *white noise* $Z_n(0),..,Z_n(N\text{-}1)$
   - Randn-function
2. Shape with desired power spectrum: $Z(k)=\text{sqrt}(G(2\pi k/N)\ Z_n(k));$ $k=0,..,N\text{-}1$
3. Inverse FFT: $z(n) = \text{ifft}(Z)$

→ $<|Z(w)|^2> = G(w)|Zn(w)|^2 = G(w)$; for $w= 2\pi k/N$

Power spectrum for z(n):
(smoothed version of G(w))

$$G_Z(\omega) = \frac{1}{2\pi}\int_{-\pi}^{\pi} d\lambda \left|W(\lambda)\right|^2 G(\omega-\lambda)$$

Autocorrelation function:

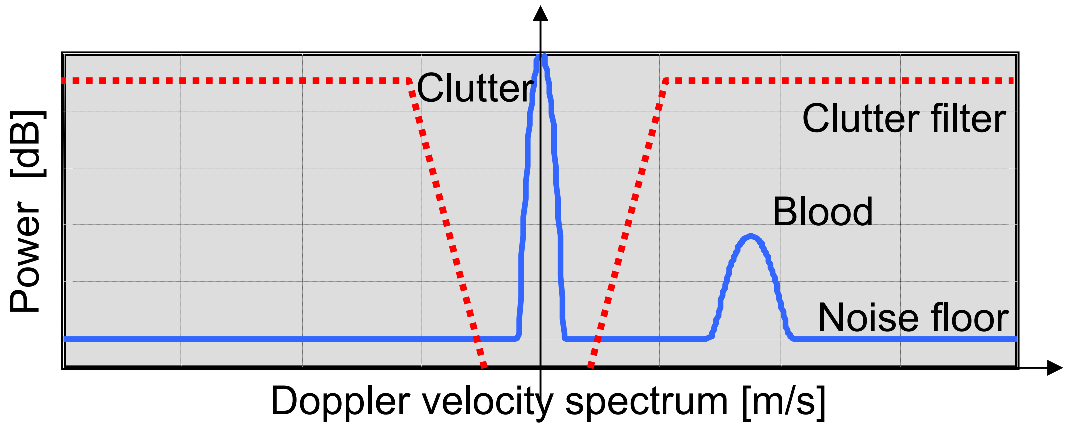
$$R_z(m) = Tri(m)\cdot R(m)$$

Doppler signal model

- The Doppler spectrum may consists of three components, clutter *c*, blood *b,* and thermal noise *n*

$$\mathbf{x} = \mathbf{c} + \mathbf{n} + \mathbf{b} \qquad \mathbf{x} = [x(1),\ldots,x(N)]^T$$

- Typical clutter/signal level: 20 – 80 dB
- Signal from blood is characterized by a complex Gaussian process



Simplified Doppler signal model

- Gaussian correlation function

$$R(m) = 10^{SNR/10} e^{-\frac{1}{2}\left(\frac{m}{\sigma_2}\right)^2} e^{i2\pi f_d m} \qquad G(\omega) = 10^{SNR/10}\sqrt{2\pi}\sigma_2 e^{-\frac{1}{2}(\omega-\omega_d)^2 \sigma_2{}^2}$$

$$\sigma_2 = T_r / T_{prf} \qquad T_r = L/v_r = \frac{N_p \cdot c / f_0}{v_r}$$

L=pulse length in [m]

- White noise is delta-correlated

$$R_n(m) = \delta(n-m)$$

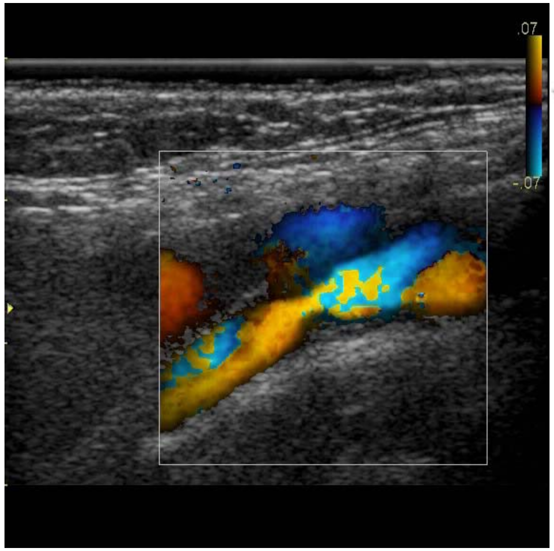
Simulation example: Color Flow Imaging - estimator properties

Data acquisition → **Clutter filtering** → **Parameter estimation** → **Display**

Color Flow Imaging displays a *color coded map* of the *mean axial blood velocity* in a 2-D or 3-D region of interest

Acquisition and processing steps

- CFI data acquisition
- Clutter filtering (wall filtering)
- Doppler parameter estimation
- Display



Color Flow Imaging of a carotid artery stenosis

The autocorrelation estimator

In the autocorrelation approach, the following mean power and Doppler frequency estimators are used:

$$\hat{P} = \hat{R}(0) = \frac{1}{N}\sum_{n=0}^{N-1} z(n)z(n)^{*} = \frac{1}{N}\sum_{n=0}^{N-1} \left|z(n)\right|^{2}$$

$$\hat{f}_d = \frac{\angle \hat{R}(1)}{2\pi} = \frac{1}{2\pi}\angle\left[\frac{1}{N-1}\sum_{n=0}^{N-1} z(n)^{*} z(n+1)\right], \quad -0.5 \leq \hat{f}_d \leq 0.5$$

In other words: The power, mean frequency of the Doppler spectrum can be found using magnitude and phase estimates of the correlation function at lags 0 and 1

Reference: C. Kasai, *Real-Time Two-Dimensional Blood Flow Imaging Using an Autocorrelation Technique*, IEEE Transactions on Sonics and Ultrasonics, vol. 32 pp. 458-464, 1985

Exercise 1

- Make a matlab script to generate the Doppler signal from blood moving at a given velocity

- Perform the autocorrelation approach to estimate the power and velocity

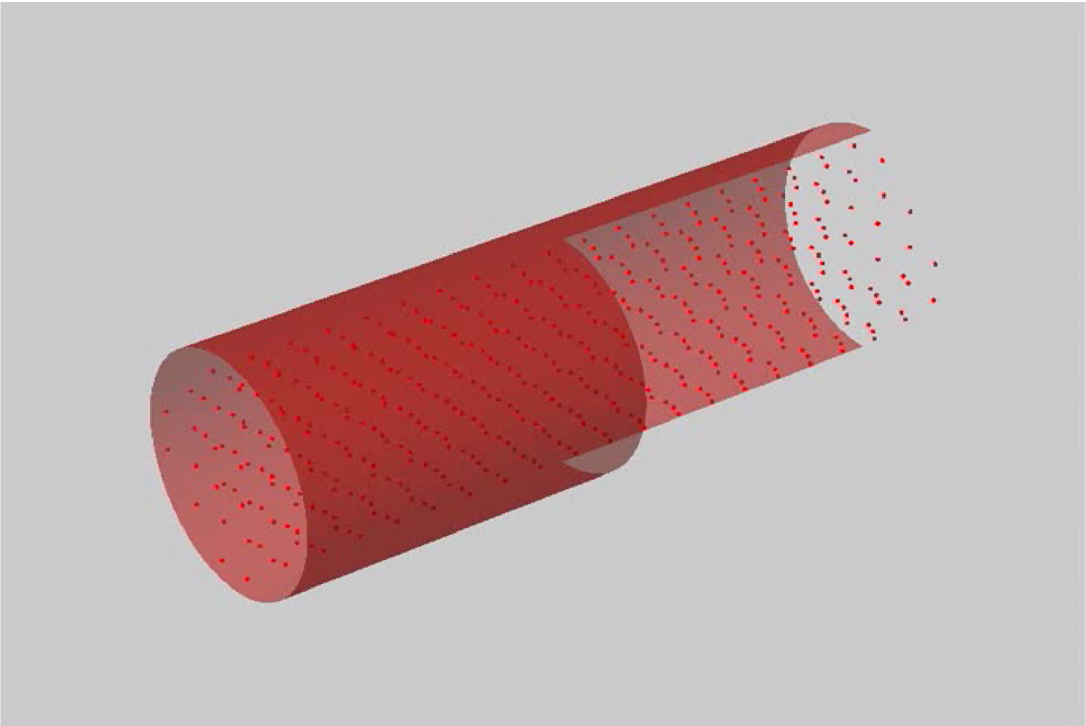
- Do the same for *N* signal realizations
  - Is the autocorrelation approach unbiased?
  - How does the variance vary with bandwidth (1/transit time)

- Add noise with sigma^2=1
  - SNR=30/5: Is the autocorrelation approach unbiased?

Simulation of blood flow in Field II

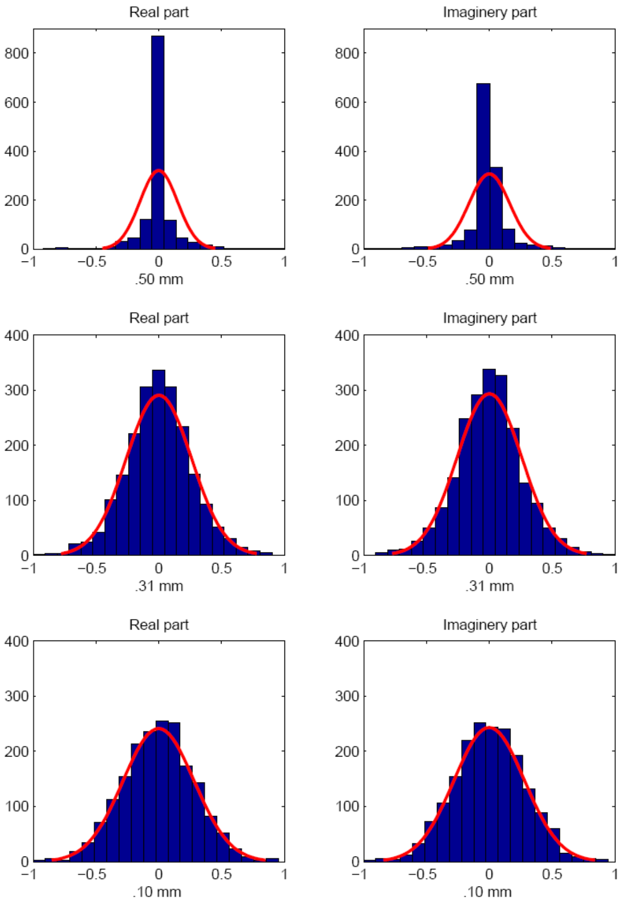
Imaging blood flow in Field II

- Blood is basically modeled as a large collection of point scatterers moving at given velocities and angles



How many point scatterers are needed?

- **Measure:** Is the received signal amplitudes Gaussian distributed?
- Approximately 10 scatterers per resolution cell needed
- Resolution cell defined as radial sample volume x beam widths



Advantages of Field II for blood flow simulations

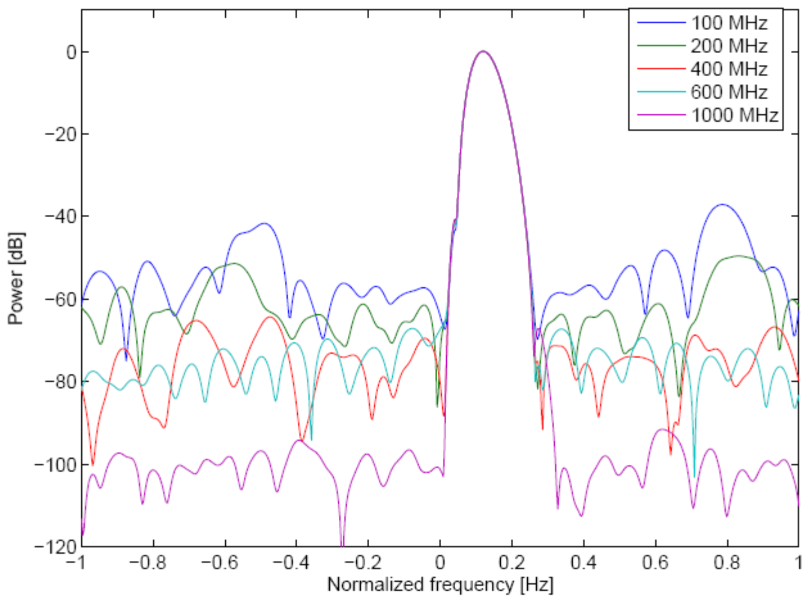
- Realistic scan sequencing can be imposed
  - Each beam is formed separately
  - Pw-Doppler, CFI, beam interleaving, …
- Realistic point spread functions
  - Spatially varying
- 3-D phantoms and simulations can be performed with relative ease
- Arbitrary and realistic transducers can be modeled
  - 1-D, 2-D arrays, annular, etc…
- Simulation times are actually not that high
  - Incorporating all tricks…

Limitations of Field II for blood flow simulations

- All the fundamental Field II limitations
  - Linear propagation
  - No reverberations or aberrations…
- Point scatterers have no shape or inertia
  - Realistic properties of blood not incorporated
  - Not valid for very low or high hematocrit values
- Rayleigh scattering properties → $P \sim f^4$
  - A point scatterer does not exhibit frequency dependent scattering
  - Can be produced by filtering the received signal

Simulation accuracy

- Sampling frequency
  - Avoid phase jitter due to simulations inaccuracies
  - 100 – 200 MHz sufficient
  - Simulation time goes linear with sampling frequency

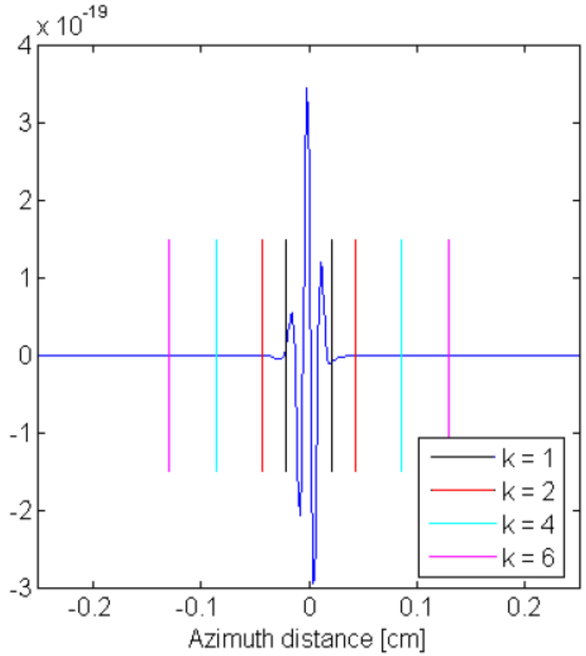


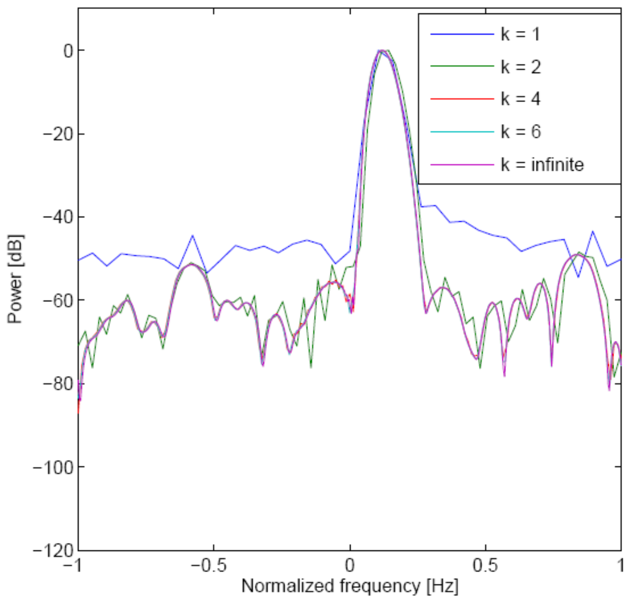
Sampling frequency	Simulation time	Normalized simulation time
100 MHz	8 min	0.574
200 MHz	14 min	1.00
400 MHz	21 min	1.50
600 MHz	29 min	2.07
1000 MHz	48 min	3.42

- Element subdivision
  - As for all Field II simulations
  - Every point scatterer should be in the far field of each mathematical element

Increasing simulation speed

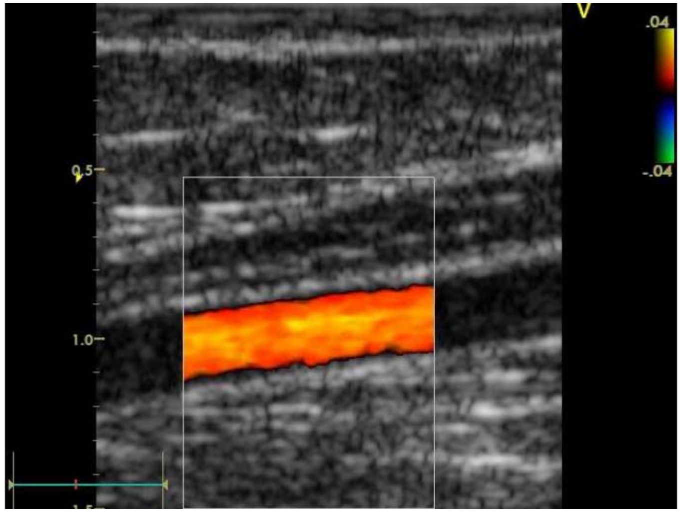
- Exclude scatterers outside a region around the given beam



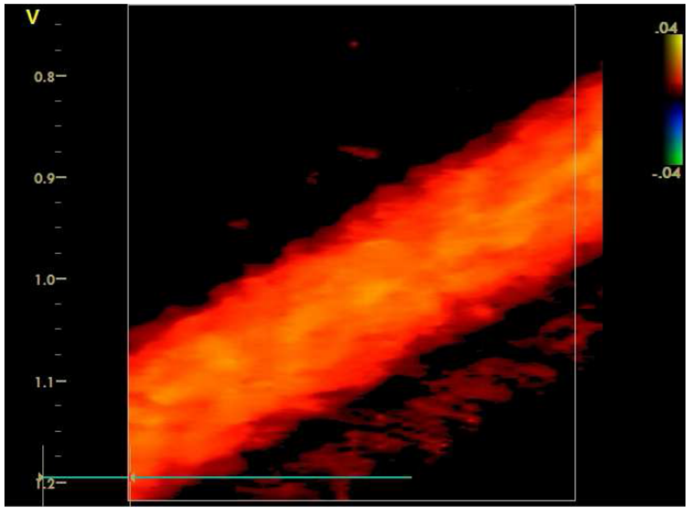


Color Flow Imaging simulation

- Simulation time: ~1 hour
  - Packet size = 10, marginally sampled (Rayleigh)



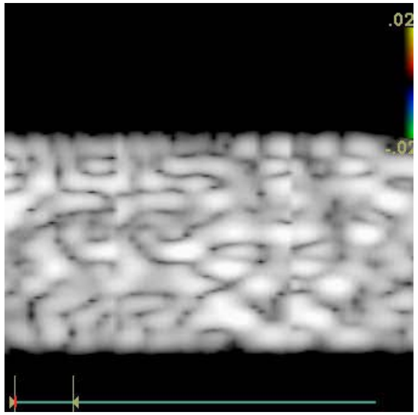
In vivo (radial artery)



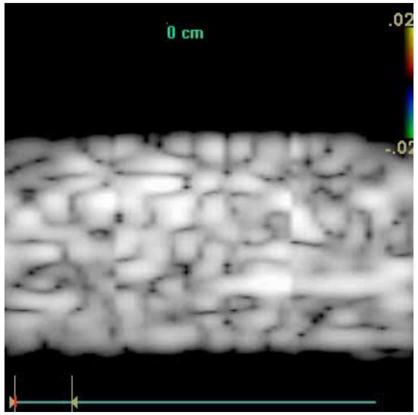
Simulated image

Speckle correlation

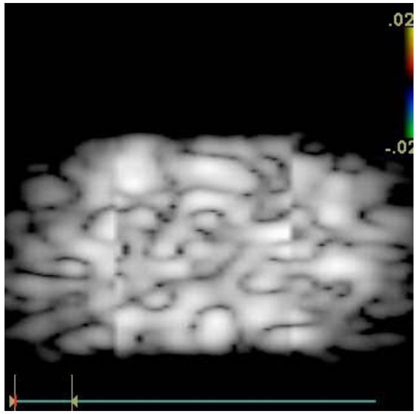
- Varying out-of-plane angle



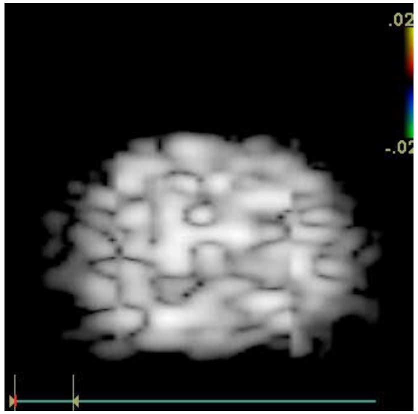
10 deg



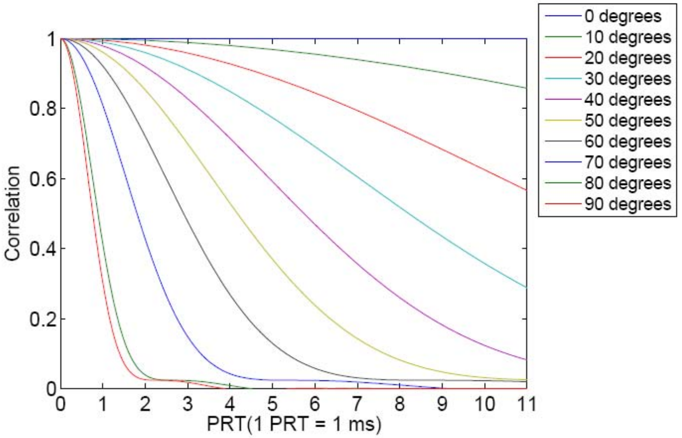
30 deg



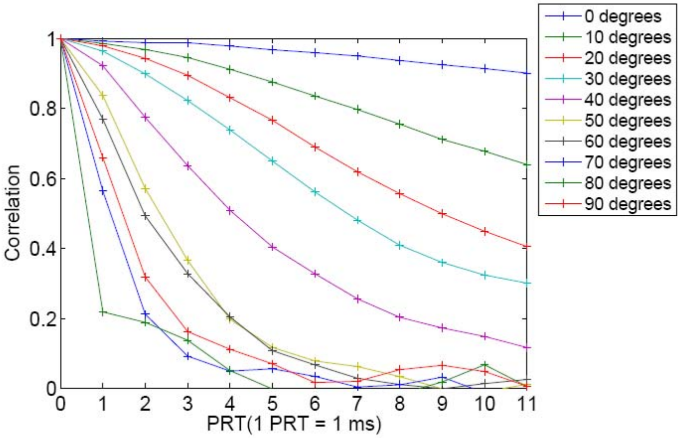
50 deg



70 deg



(a)



Exercise 2

- Setup Field II for PW-Doppler acquisition
  - Single beam position (x=0), multiple firings at a given PRF
  - The dynamic phantom should be a single point scatterer moving at a velocity and angle through the focal point of the beam
- Field II specifics
  - Use the *calc_scat* function
  - Align the received beams in time using the start_time returned for each beam
  - Form the analytic Doppler signal: z(n) = RF + i*Hilbert(RF), then downmix to baseband for the final complex form
- Plot the Power spectrum of the final Doppler signal

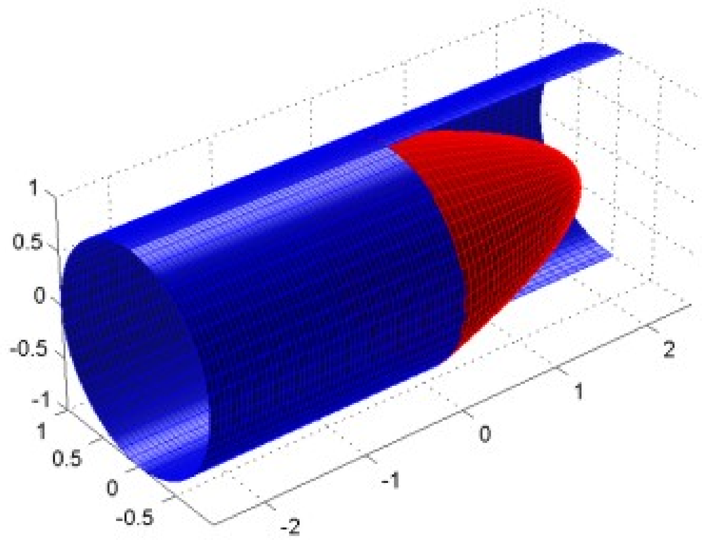
Analytic blood flow models

Poiseuille flow model

- Established steady laminar flow in long cylindrical pipes is sometimes referred to as *Poiseuille flow*

- The fluid moves in a series of concentric shells for which the fluid viscosity leads to a parabolic velocity profile. At the vessel wall the velocity is zero

- When p=2 → parabolic flow, p=inf → blunt (rectangular)



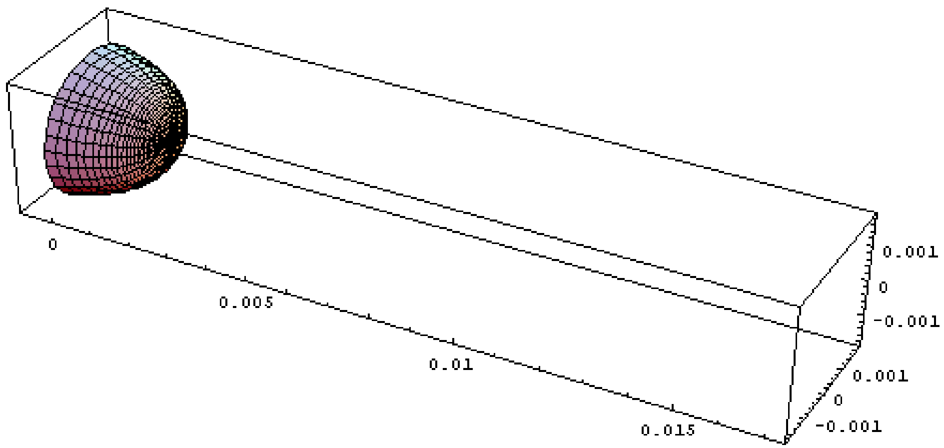
$$v(r) = v_{max}\left(1 - \frac{r^2}{R^2}\right)^p, \quad p = 2$$

Womersley pulsatile model

- Similar assumptions as for Pouseuille flow, but solved for a sinusoidal pressure gradient
  - Laminar flow
  - Constant pressure gradient in space
  - Newtonian fluid
- Womersley solved for flow resulting from a sinusoidal pressure gradient input
- For a measured pressure or flow curve, the total flow or velocity field can be predicted by a Fourier decomposition of the input curve

Womersley pulsatile model

- The total solution is obtained by summing the contribution for each harmonic component of the input curve, resulting in **a time-varying velocity profile**



Single sinusoidal flow input

$$Q\cos(\omega t-\phi)$$

$$U(y,t)=\frac{1}{\pi R^2}Q|\psi|\cos(\omega t-\phi+\chi)$$

$$\psi=\left(\frac{\tau J_0(\tau)-\tau J_0(y\tau)}{\tau J_0(\tau)-2\tau J_1(\tau)}\right) \qquad \tau=\alpha i^{3/2}$$

(Jx: Bessel function of order 0/1)

Total solution

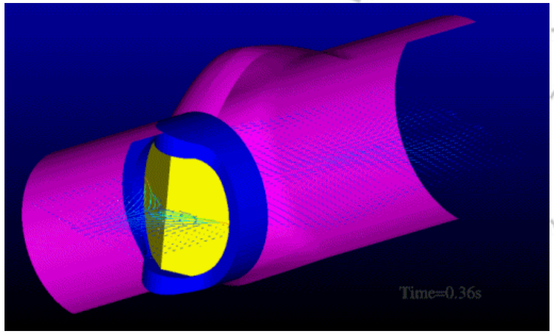
$$V(t)=V_0+\sum_{p=1}^{\infty}V_p\cos(p\omega t-\phi_p) \qquad U(y,t)=\left\{2V_0(1-y^2)+\sum_{p=1}^{\infty}V_p|\psi|_p\cos(p\omega t-\phi_p+\chi_p)\right\}$$

Computational blood flow models

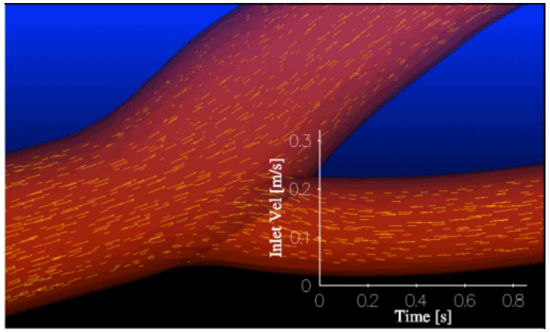
Computational fluid dynamics

- **Computational fluid dynamics (CFD)** uses numerical methods to analyze fluid flows
- Input to CFD is a **3-D geometrical model** and flow and/or pressure inlet and outlet conditions
- The flow domain is divided into **a grid of control volumes** where flow variables are calculated and propagated for each grid point

CFD simulation of flow around artificial heart valves

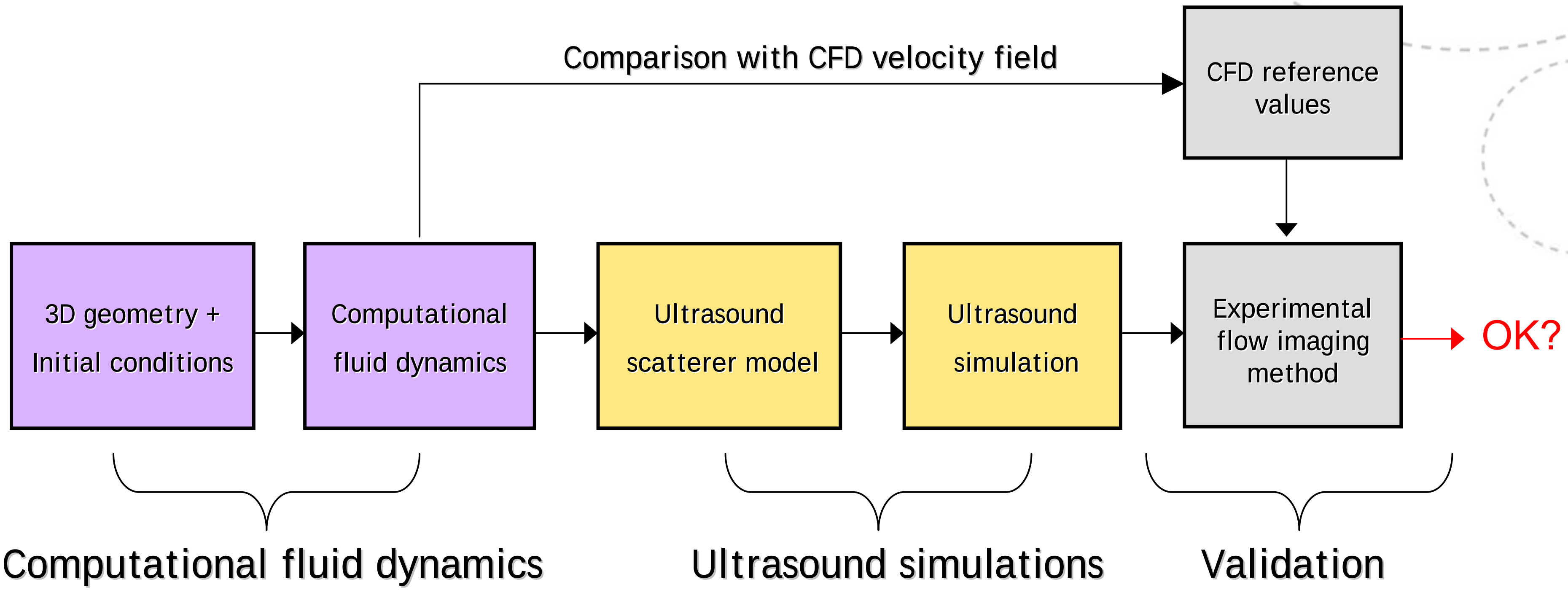


CFD simulation of flow in a carotid artery



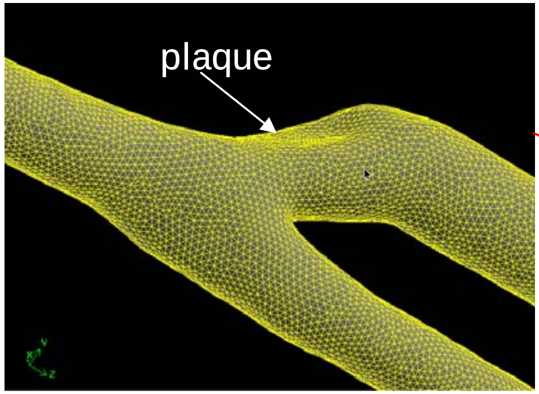
Animations: http://www.realcfd.com

Validation of new algorithms using computational fluid dynamics

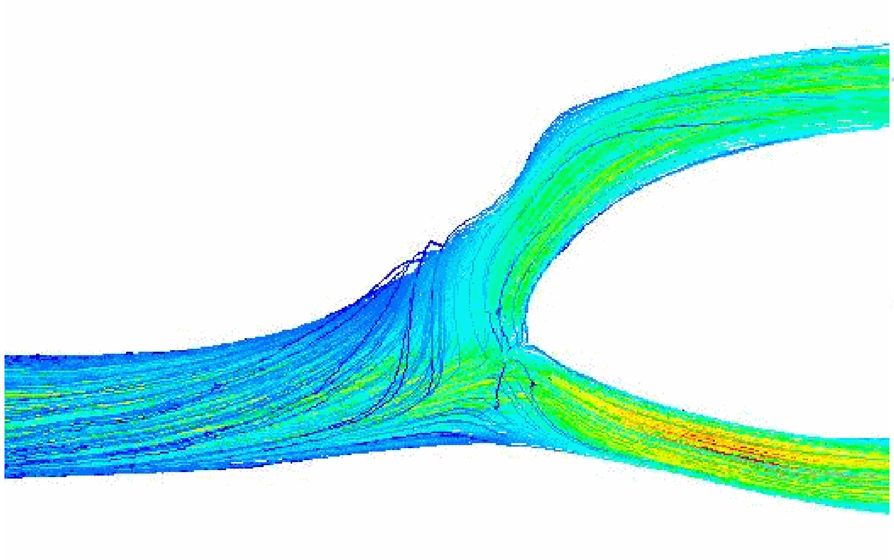


Input boundary conditions

3D reconstruction carotid

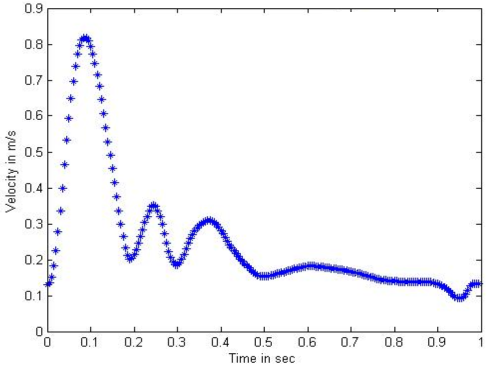


CFD simulation



Boundary conditions

Inlet velocity profile



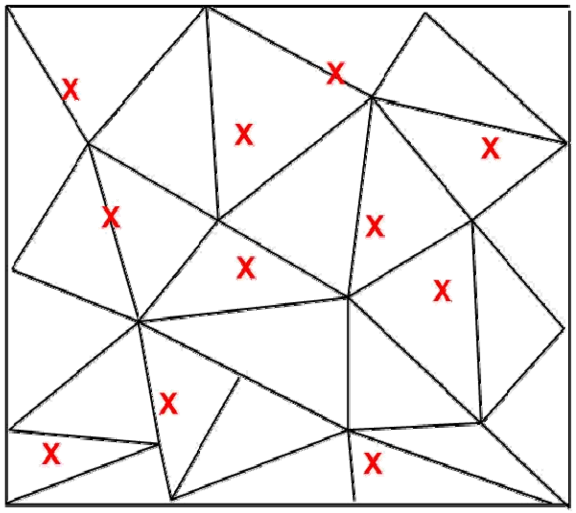
Outflow division: 45% external-55% internal

Creation of CFD phantom

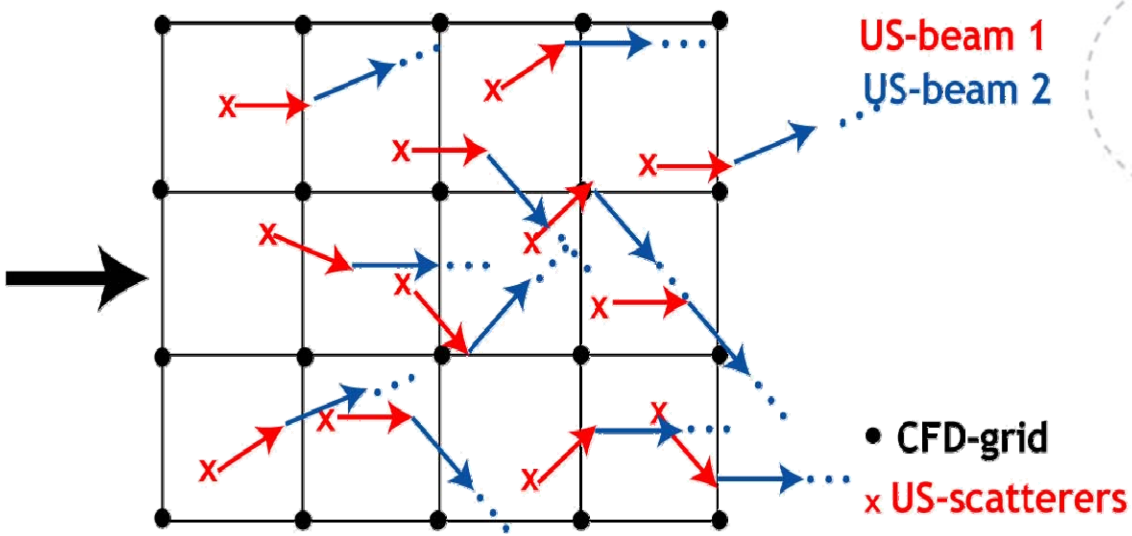
3D Spatial interpolation:

Matlab procedure: direct 3D interpolation from unstructured CFD grid not possible

Unstructured CFD grid



Hypersurface fitting to structured grid

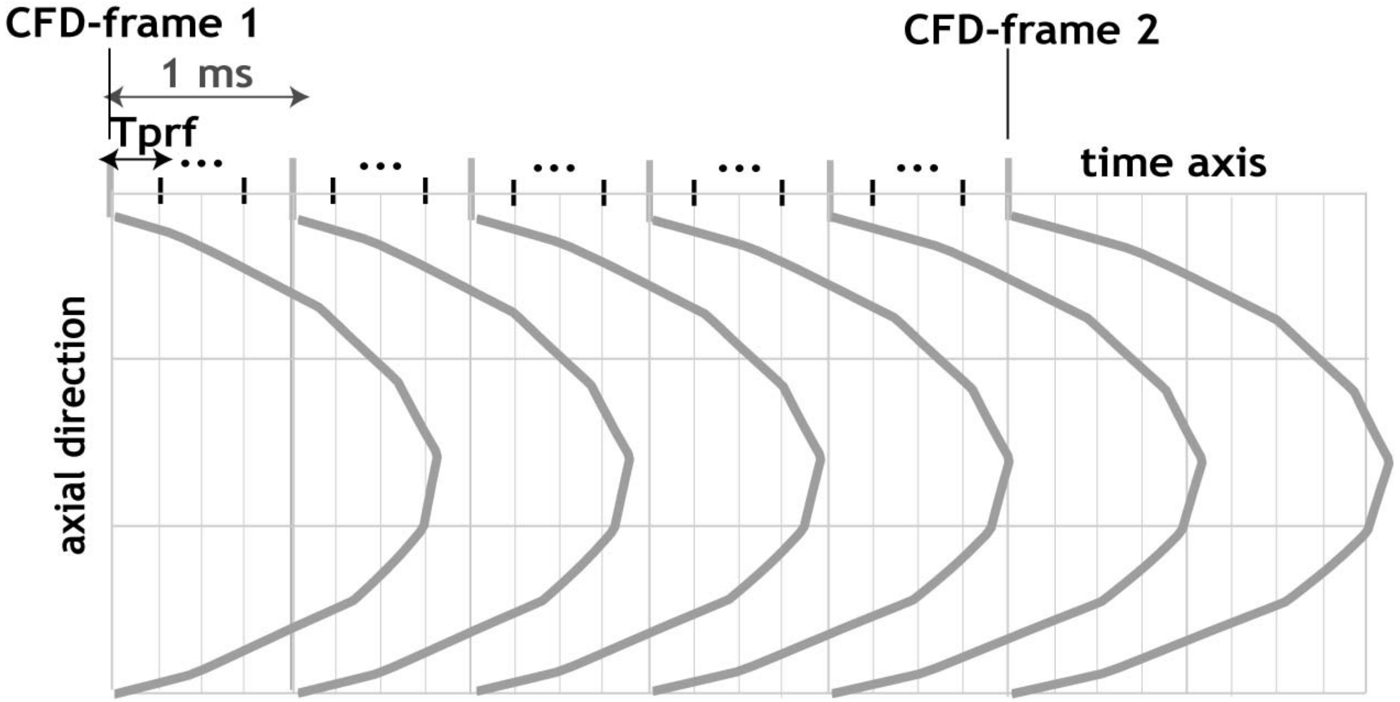


Using structured grid, 3D interpolation to random points (x) is possible!

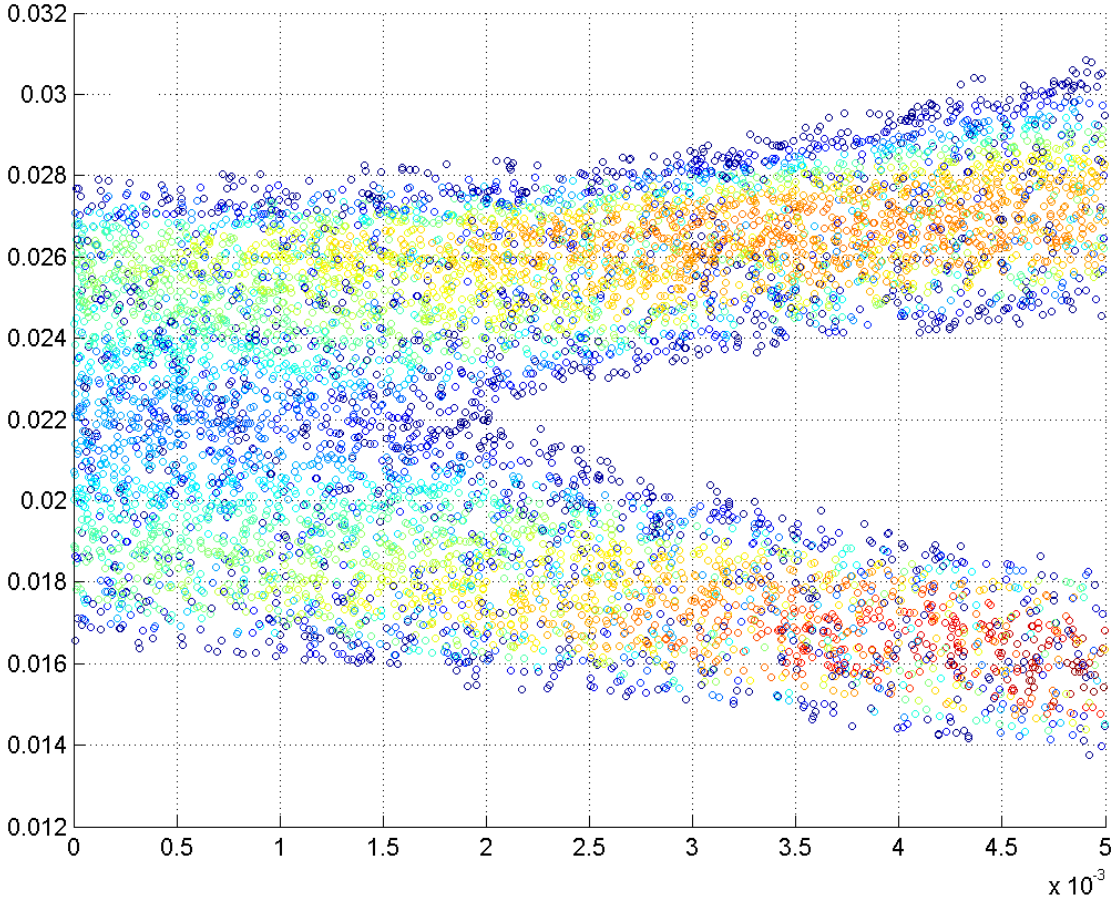
Creation of CFD phantom

Temporal interpolation:

CFD time scale >> ultrasound time scale



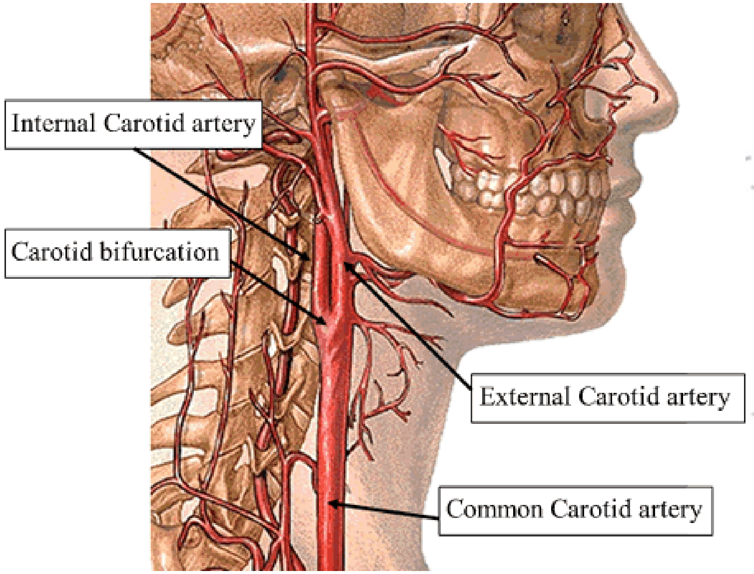
Creation of CFD phantom



$\Delta T = 1/PRF$ PRF = pulse repetition frequency

Example: A carotid bifurcation

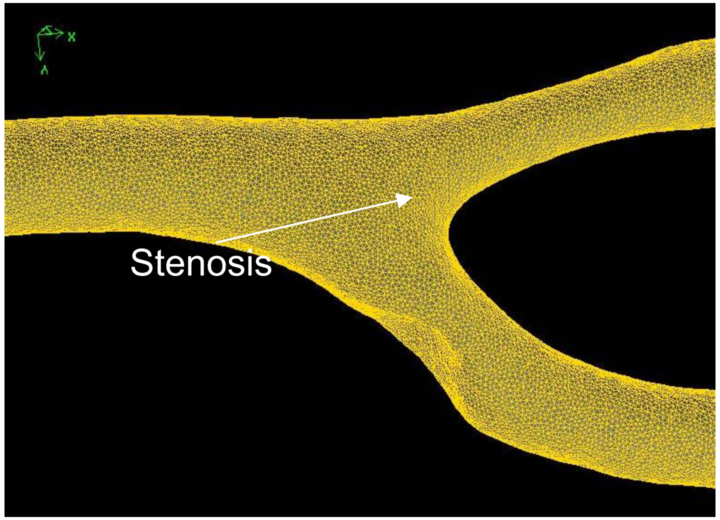
- An in vitro silicon model was made based on MR images (1.5T) of a healthy subject
- Plaque was artificially inserted into the internal carotid to create a stenosis
- The silicon model was scanned by CT, and a CFD simulation mesh created from this volume data
- Flow inlet conditions were extracted from ultrasound Doppler measurements



Carotid bifurcation modeling

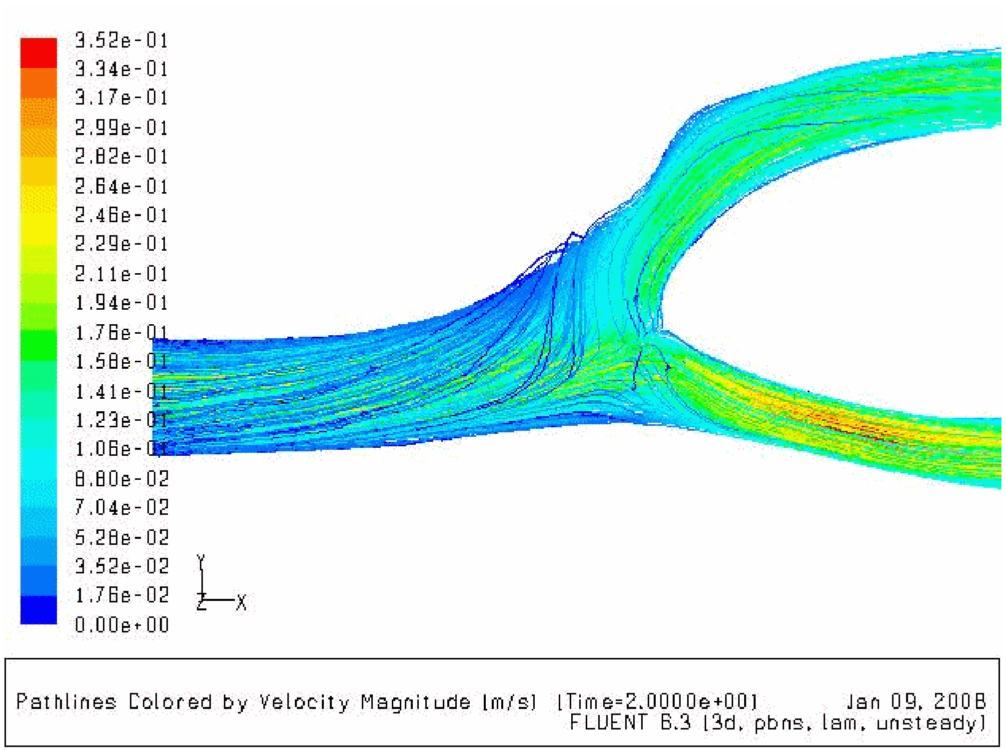
- A 3-D model of a carotid artery bifurcation was made from silicon model based on MR-images of a healthy subject. Eccentric plaque was added artificially, creating a stenosis.

Silicon model

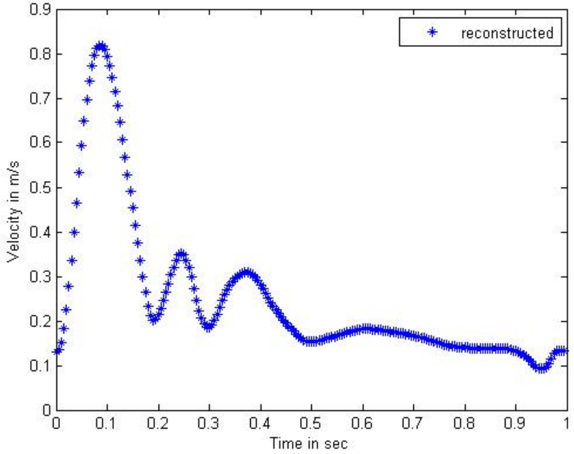


CFD simulation mesh

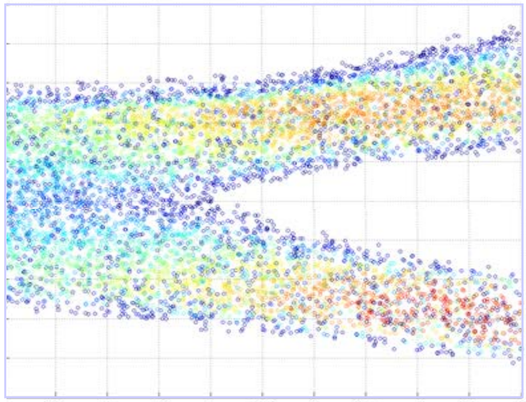
Carotid CFD simulations



Dynamic CFD simulation, one cardiac cycle

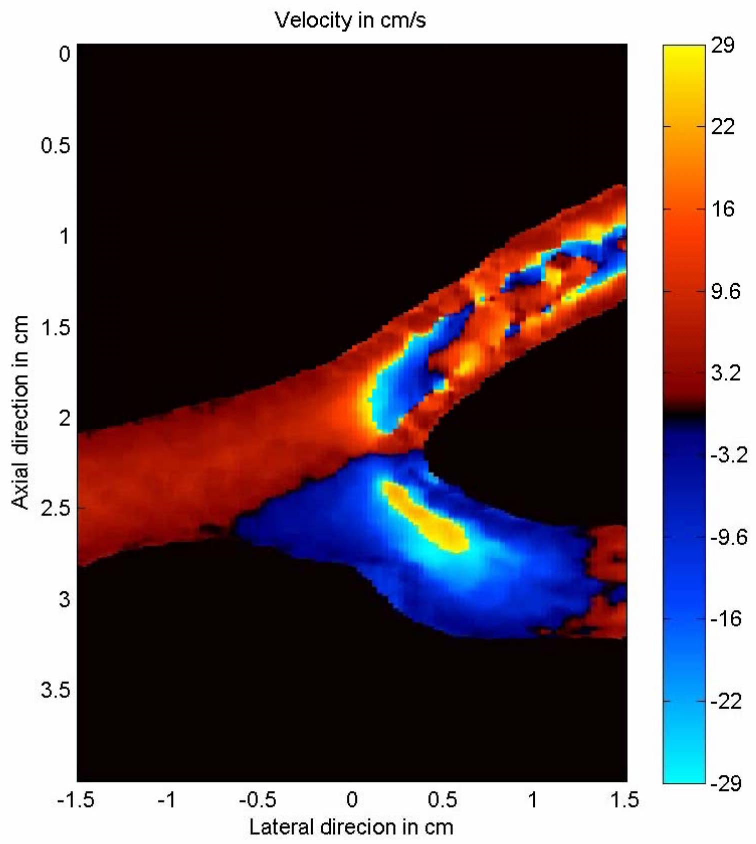


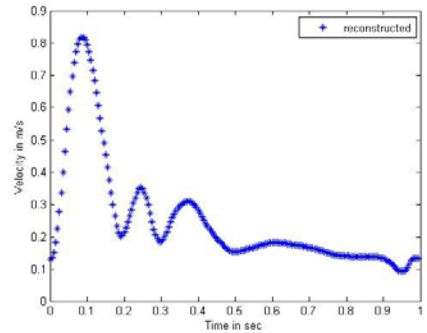
Velocity curve measured by PW-Doppler



Point target phantom for ultrasound

Color flow imaging simulation





Inlet velocity curve

A complete cardiac cycle was simulated

Simulation times

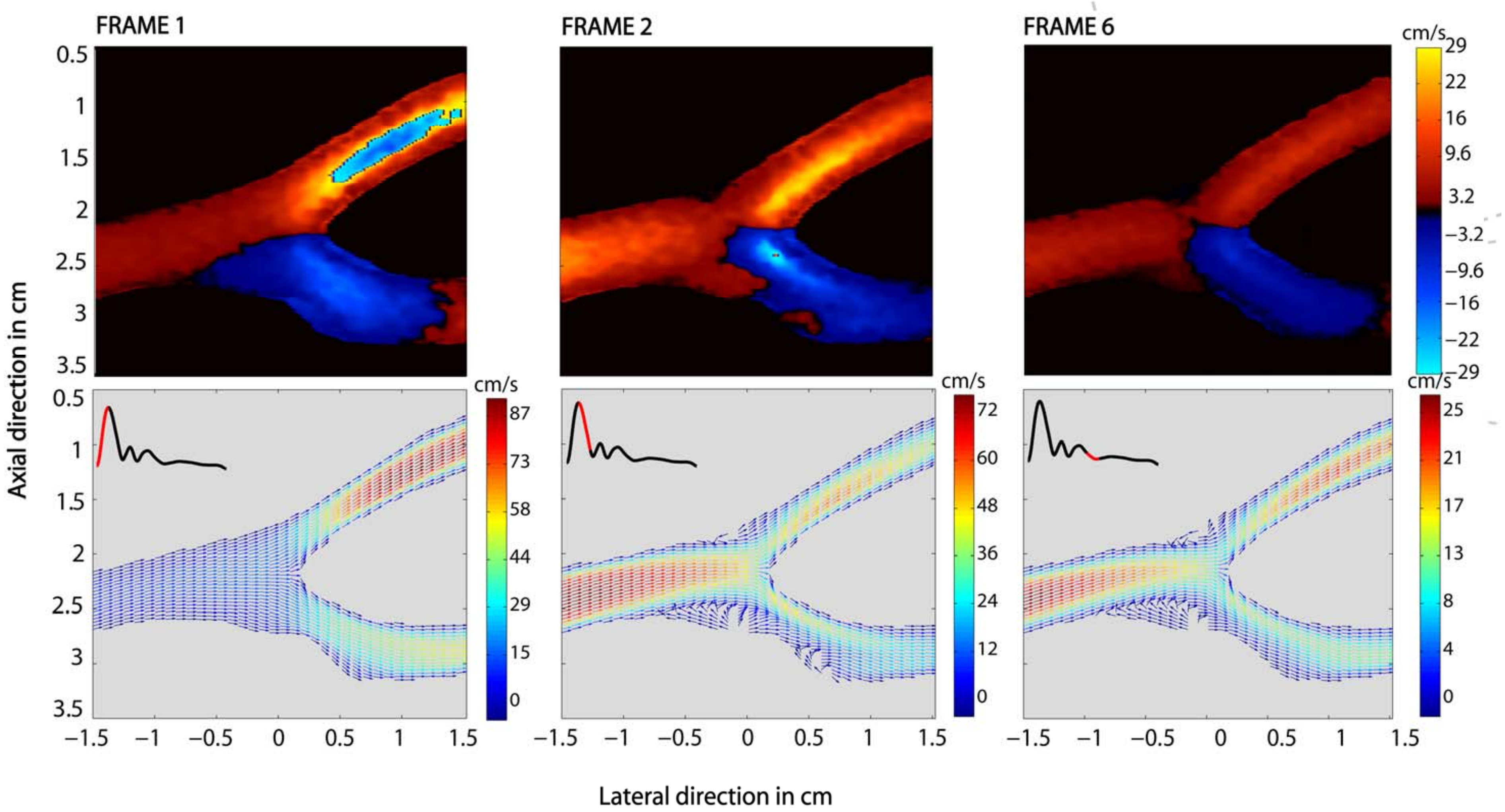
CFD: 220 hours

(2.4 GHz Intel Core Duo)

Ultrasound: 8 hours per frame, 11 frames in total

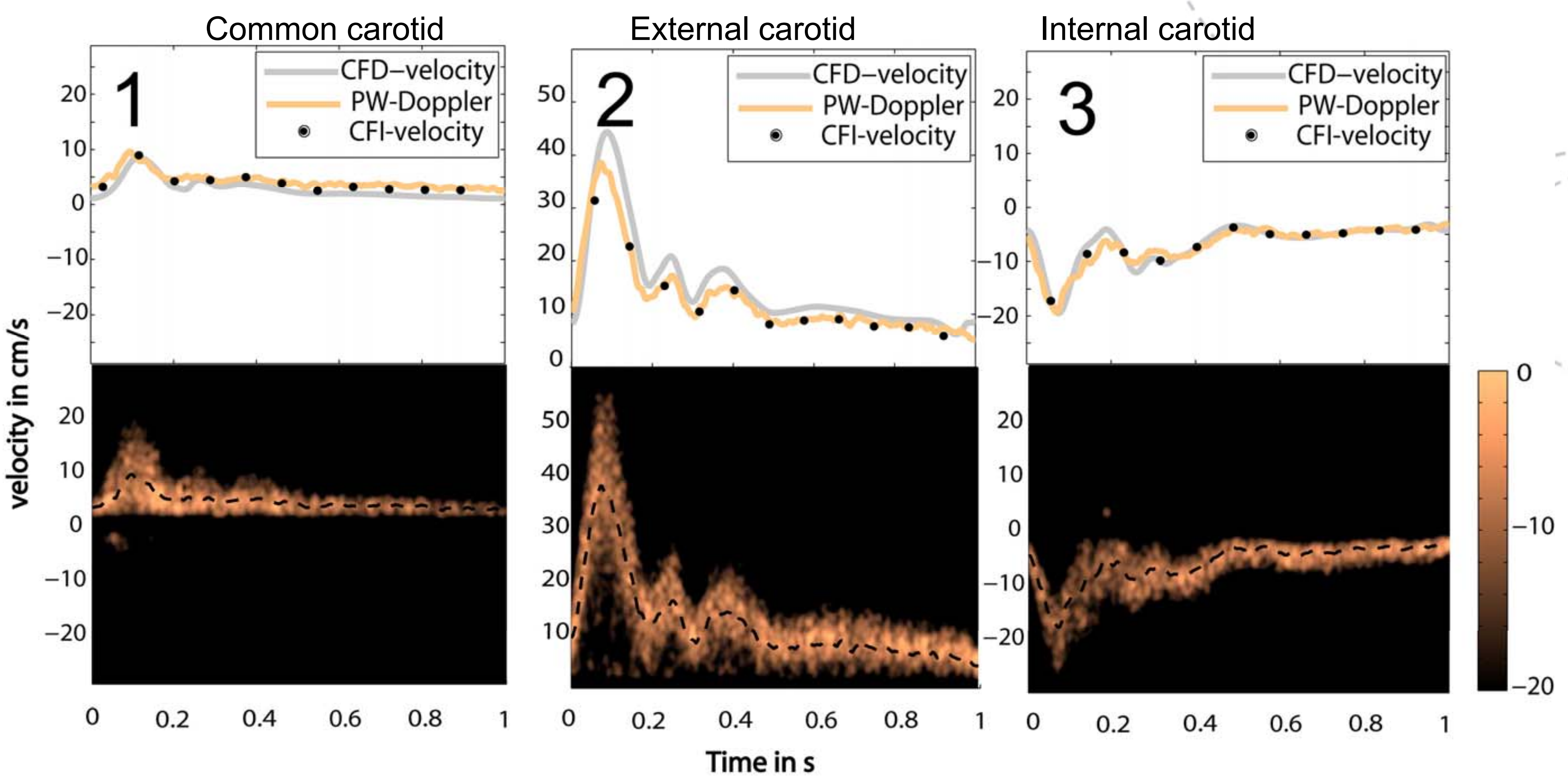
(3.4 GHz Pentium 4)

Color flow imaging simulations



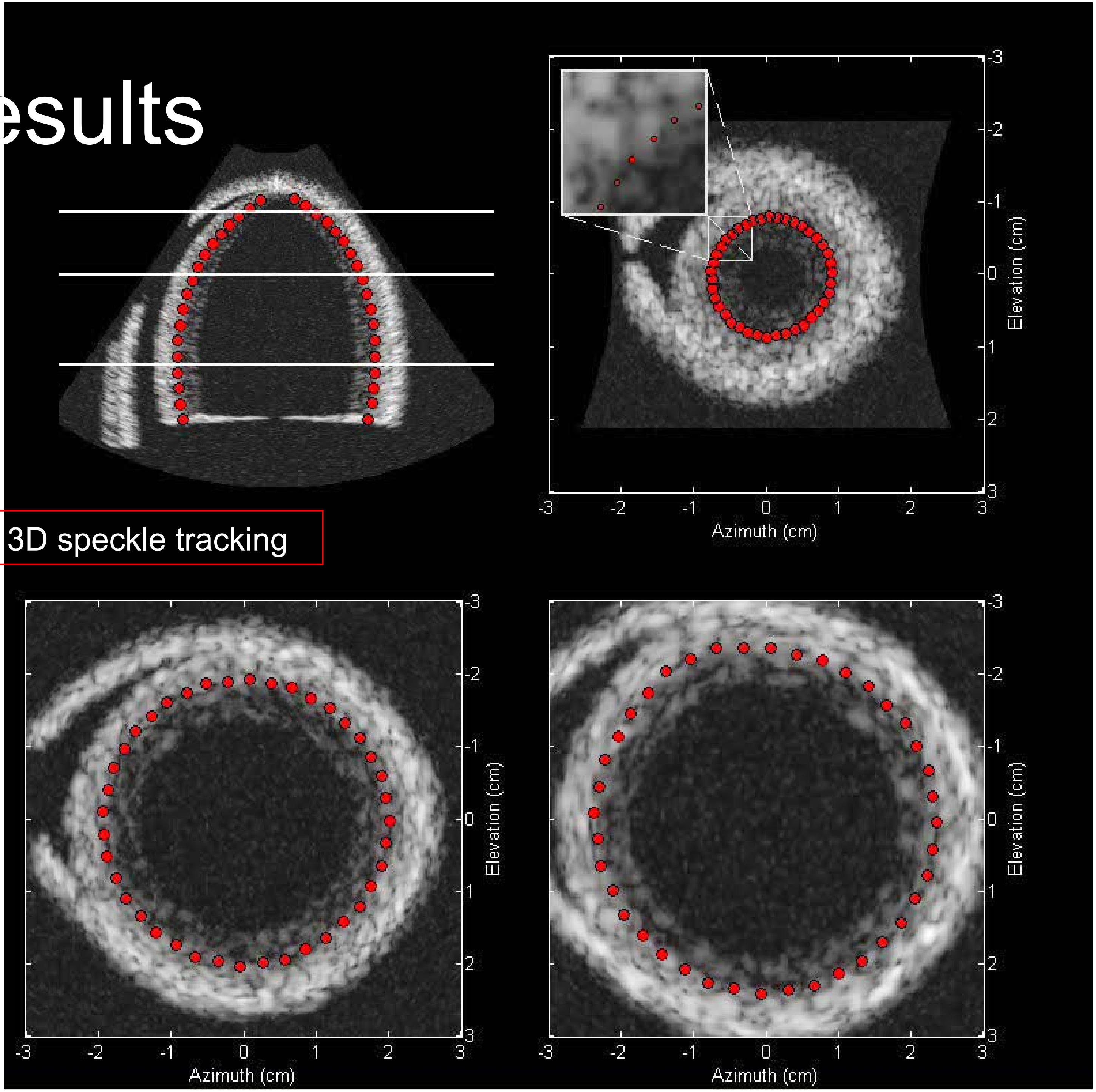
Simulations can reveal the shortcomings of Doppler methods

PW-Doppler simulations



PW-Doppler simulations from the three branches of the carotid bifurcation

Results



3D speckle tracking

Testing automated
volume measurements